

# FiveThirtyEight's May 7, 2021 Riddler

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This week's riddler, courtesy of Matt Yeager, is a puzzle about a game:

**Question 1.** *Three of Matt's students - Players A, B and C - are engaged in a game of veinte. In each round, players take turns saying numbers in order (Player A, then B, then C, then A again, etc.). The first player to go says the number "1." Each number must be either one, two, three or four more than the number said by the previous player. When someone says "20," the round is over and the next person is eliminated, with the following person beginning the subsequent round. For example, if Player A says "20," then Player B is eliminated, while Player C begins the next round by saying "1." At no point can anyone say a number greater than 20.*

*All three players want to be the winner (i.e., the only player remaining) after the two rounds. But if they realize they can't win, then they will prioritize making it to the second round.*

*Player A starts things off by saying "1." Which player will win?*

*Extra Credit: What if there are four players (A, B, C, and D)?*

Let's start with the two-player game, where I will assume that player A is the first to go and they're playing against player B. Player B can win no matter what player A does by naming 5, 10, 15, and 20 in their turns, and there is nothing player A can do to stop that.

Thus, if player B wins (names 20) the first game, then player C is eliminated, and player A starts the second game, meaning that if you win the first game you also win the second game. That tells us what happens if some numbers are named: if player A manages to name 20, then they win. If they name any number between 16 and 20, player B wins, but player A at least makes it to the second round.

What if player A names 15? Then player B cannot win, but must let player C win. Therefore, if player A names 15, not only do they not win, but they don't even make it to the second round. Similarly, if player A names 14, then player B is faced with deciding between letting player C win immediately by naming something between 16 and 18, or letting player A win by naming 15. But player B will not name 15 as that gets them eliminated in the first round, so player B will let player C win and player A will be eliminated. For similar reasons, if player A names 13 or 12, then they will be eliminated in the first round.

Now, we reach 11. If player A names 11, player B must name something between 12 and 15. At this point player C has no better option than to let player A win. Therefore, if player A names 11, they win!

And now it repeats: if player A names anything between 7 and 10, player B wins by naming 11. If player A names anything between 3 and 6, player B cannot win, but will let player C win, so player A gets eliminated in the first round. If player A names 2, then they win, and so on.

Thus, in the three-person game, player B names 2 and will win eventually.

Now we start the same analysis for the four person game: assume A names 20. Then B gets eliminated, C starts the second round, D wins the second round (and thus the whole game), and A gets eliminated in the second round.

Now, if A names 19, B has no choice but to name 20. Then C gets eliminated, D starts the next round, and A wins overall. So 19 is a winning number.

Consequently, if A names anything between 15 and 18, B will name 19 and win eventually. If B names 19, D gets eliminated in the first round, C gets eliminated in the second round, and A gets eliminated in the third round. Thus, naming any number between 15 and 18 gets A eliminated in the third round.

Now, if A names 14, B must let C name 19, which forces D to name 20 and eliminate A. Similarly, if A names anything between 11 and 13, B would rather let C name 19 than let D name 19, meaning A gets eliminated in the first round if they name anything between 11 and 14.

And now we reach 10. If A names 10, B has to name something between 11 and 14. Then C's best option is to let D name 19, which forces A to name 20, eliminate B, and get eliminated in the second round.

If A names 9, B would much rather name 10 than anything between 11 and 13 as that keeps B alive for another round. Thus, if A names 9, B eventually names 20, making A the winner of the whole game! Thus, 9 is another winning number, and the pattern repeats: 5 to 8 lose in the third round, and 1 to 4 lose immediately.

Thus, if player A names 1, player B names 5, and player C names 9 and wins. Therefore, in the four-person game, player A gets eliminated in the first round, player D gets eliminated in the second round, player B gets eliminated in the third round, and player C wins. Player A just can't seem to catch a break!