FiveThirtyEight's May 20, 2022 Riddler

Emma Knight

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This week's riddler is about black hole photography:

Question 1. *Assuming the accretion disk of a black hole is equally likely to be in any plane, what is the probability of it being within* 10 *degrees of perpendicular to us, thereby resulting in a spectacular image?*

To think about the geometry of the situation, the accretion disk is within 10 degrees of being perpendicular to us if and only if the normal vector is within 10 degrees of being parellel to us (note that there are two normal vectors, but choosing a vector that is at most 90 degrees off being parellel is unique up to a set of measure zero). Since there is a hemisphere of possible normal vectors (see previous parenthetical), one merely needs to compute the area of this ten degree cap, and divide it by 2π to get the probability.

This cap is easily parameterized by $f(u,v) = (\cos(u)\sin(v), \sin(u)\sin(v), \cos(v))$ for $u \in [0, 2\pi]$ and $v \in [0, \pi/18]$. Computing the area is now standard: $f_u = (-\sin(u)\sin(v), \cos(u)\sin(v), 0)$ and $f_v = (\cos(u)\cos(v), \sin(u)\cos(v), -\sin(v))$. Then $||f_u \times f_v|| = \sin(v)$, so the area is $\int_0^{2\pi}\int_0^{\pi/18} \sin(v)dvdu$. This is a very straightforward integral that works out to $2\pi(1-\cos(\pi/18))$. Thus, the probability of the disk image being this nice is $1 - \cos(\pi/18) \approx .01519$.